

# **METRO EAST EDUCATION DISTRICT**

## **COMMON PAPER**

**GRADE 12**

**MATHEMATICS P2  
SEPTEMBER 2018**

**MARKS: 150**

**TIME: 3 hours**

**This question paper consists of 12 pages, 1 information sheet and an  
ANSWER BOOK of 20 pages.**

Please turn over

**INSTRUCTIONS AND INFORMATION**

**Read the following instructions carefully before answering the questions.**

1. This question paper consists of 11 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided. Write your name and class in the space provided and hand it in.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
5. Answers only will NOT necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, round off answers to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
10. An information sheet with formulae is included at the end of the question paper.
11. Write neatly and legibly.

**QUESTION 1**

90 learners wrote a test out of 60, and their results are shown in the table below.

<b>Mark</b>	<b>Frequency</b>
$0 < x \leq 10$	2
$10 < x \leq 20$	9
$20 < x \leq 30$	21
$30 < x \leq 40$	32
$40 < x \leq 50$	19
$50 < x \leq 60$	7

- 1.1 Complete the cumulative frequency table in the **answer book**. (2)
- 1.2 Draw a cumulative frequency curve (ogive) for the data above. (3)
- 1.3 An award was given to all learners who obtained above 45 out of 60.  
How many learners received an award? (2)
- [7]**

**QUESTION 2**

- 2.1 Two learners, Dalvon and Chumani, exercised hard one afternoon. They both recorded their pulse rates,  $P$ , and time,  $t$  minutes, after they were done exercising. Dalvon's results are shown in the table below.

<b>Time in minutes (<math>t</math>)</b>	0,5	1,0	1,5	2,0	3,0	4,0	5,0
<b>Dalvon's pulse rate (<math>P</math>)</b>	125	113	104	95	82	85	72

- 2.1.1 Calculate the equation of the least squares regression line for Dalvon's results in the form  $y = a + bx$ . (3)
- 2.1.2 Use your equation of the least squares regression line to estimate Dalvon's pulse rate after 2,5 minutes.  
Give your answer to the nearest whole number. (2)
- 2.1.3 The equation of the least squares regression line for Chumani's results is  $y = 116,79 - 5,61x$ . The fitter you are, the quicker your pulse rate returns to normal. Who is fitter of Dalvon and Chumani? Give a reason for your answer. (2)

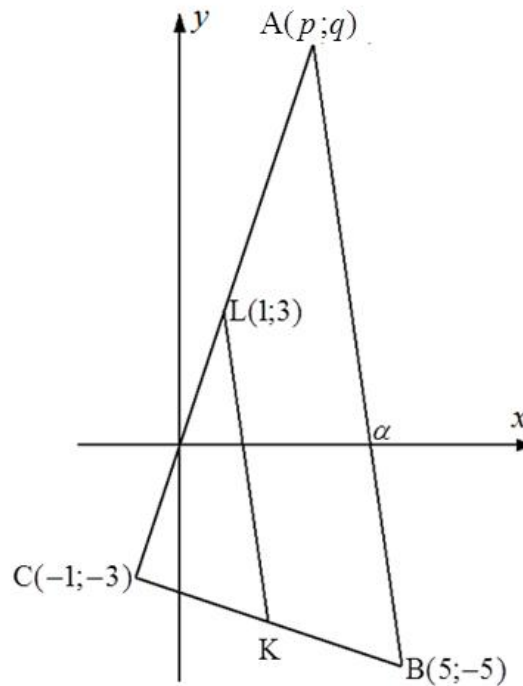
2.2 Three learners cleaned sea birds after an oil spill. They cleaned three different types of birds. The summary of the number of birds they cleaned, as well as the mean and standard deviation of the total number of birds are given in the table below.

	Number of birds by type			Total number of birds cleaned	
	Sea gull	Penguin	Cape Cormorant	Mean mass in kilograms	Standard deviation
<b>Andrew</b>	2	3	6	1,06	0,42
<b>Bongi</b>	6	2	8	0,77	0,27
<b>Chad</b>	1	0	1	1,50	0

- 2.2.1 What was the mass of the sea gull that Chad cleaned?  
Give a reason for your answer. (2)
- 2.2.2 An award was given to the person who cleaned the greatest mass of birds. Who received the award? (3)
- [12]

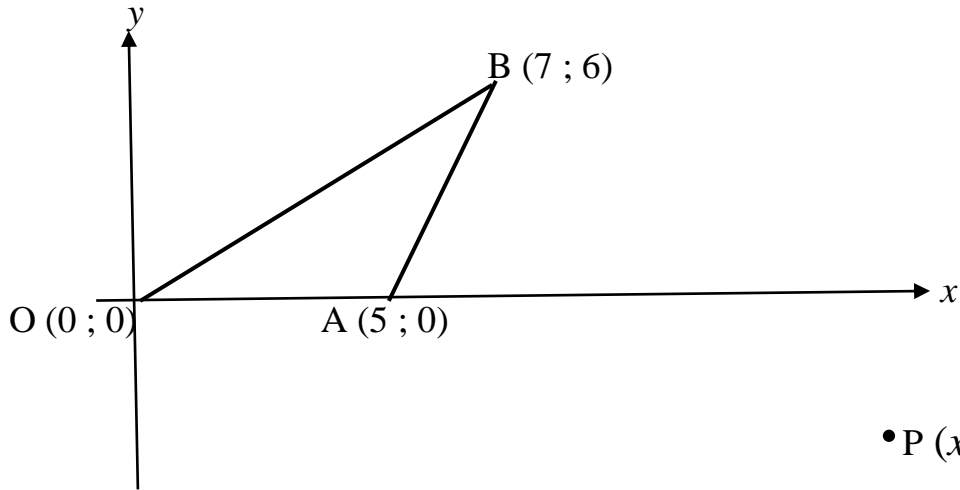
**QUESTION 3**

3.1 In the diagram A ( $p; q$ ), B ( $5; -5$ ) and C ( $-1; -3$ ) are vertices of  $\triangle ABC$ . K is the midpoint of BC and L ( $1; 3$ ) is the midpoint of AC.



- 3.1.1 Determine the co-ordinates of K. (2)
- 3.1.2 Show that  $p = 3$  and  $q = 9$ . (2)
- 3.1.3 Calculate the size of  $\alpha$ . (2)
- 3.1.4 Calculate the size of  $\hat{A}BC$ . (4)
- 3.1.5 Calculate the length of KL. Leave your answer in surd form. (2)
- 3.1.6 Without any further calculation, write down the length of AB. Give a geometric reason for your answer. (2)

- 3.2 Three points have co-ordinates  $O(0; 0)$ ,  $A(5; 0)$  and  $B(7; 6)$ . If  $P$  is the point  $(x; y)$ , calculate the values of  $x$  and  $y$ , given that the area of  $\Delta AOP$  is  $10 \text{ units}^2$ .  $PA = PB$  and  $y < 0$ .

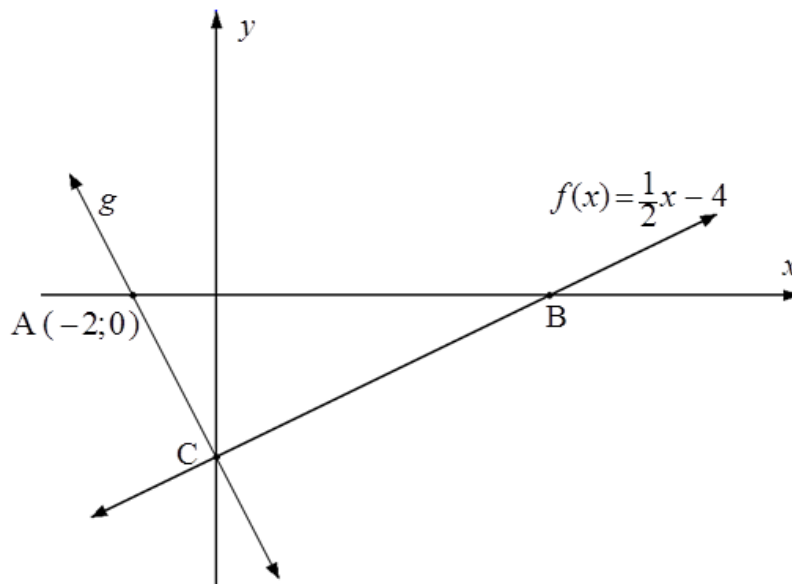


• P (x ; y) (7)

[21]

**QUESTION 4**

- 4.1 In the diagram the graphs of  $f(x) = \frac{1}{2}x - 4$  and  $g$  are represented.  $C$  is the point of intersection of  $f$  and  $g$ .  $C$  is also the  $y$ -intercept of both graphs.  $A(-2;0)$  and  $B$  are points on the  $x$ -axis.



- 4.1.1 Prove that  $BC \perp AC$  (2)
- 4.1.2 Determine the equation of the circle passing through  $A$ ,  $B$  and  $C$ . (4)
- 4.1.3 Determine the equation of the tangent to the circle through  $A$ ,  $B$  and  $C$ , in 4.1.2, at  $C$ . (3)

4.2 Determine the value of  $k$  if  $x^2 + y^2 - 8x + 10y + k = 0$  is the equation of a circle with radius 7 units. (5)

4.3 An ant walks along a path keeping a fixed distance of 3 units from the point  $Q(-1;2)$ .

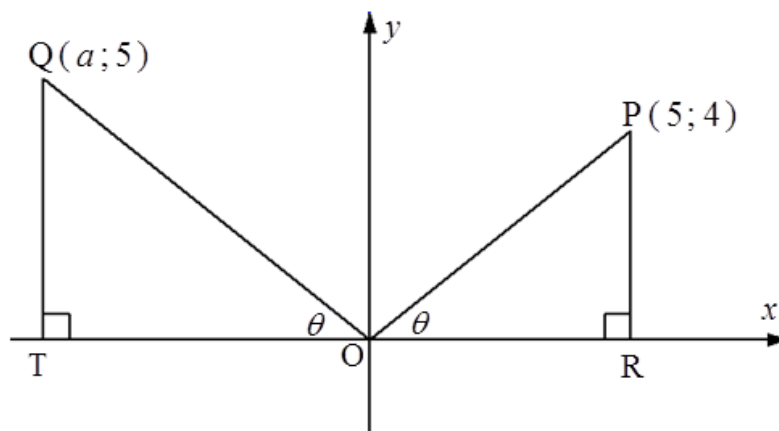
4.3.1 Draw a rough sketch on a Cartesian Plane, showing  $Q$  and the path the ant follows. (No  $x$ - or  $y$ - intercepts need to be calculated.) (2)

4.3.2 Write down the equation of the path followed by the ant. (2)  
[18]

**QUESTION 5**

**CALCULATORS MAY NOT BE USED IN THIS QUESTION.**

5.1 In the diagram below similar triangles  $\triangle OPR$  and  $\triangle OQT$  are presented.  $O$  is the origin.  $R$  and  $T$  are points on the  $x$ -axis.



Determine the following (leave answers in surd form if necessary):

5.1.1  $\sin(90^\circ - \theta)$  (3)

5.1.2 the value of  $a$ . (2)

5.2 Given:  $\frac{\cos(\theta - 180^\circ) \times \cos(\theta + 90^\circ)}{2 \tan(180^\circ - \theta)}$

5.2.1 Simplify the given expression to a single trigonometric ratio. (5)

5.2.2 For which values of  $\theta$  is the expression undefined for  $0^\circ \leq \theta \leq 180^\circ$ ? (3)

5.3 Prove that:  $\frac{\sin 2\theta}{\sin \theta} = 4 \cos \theta - \frac{\cos 2\theta + 1}{\cos \theta}$  (4)

5.4 Given:  $\frac{\sin(A-B)}{\sin(A+B)} = \frac{3}{5}$

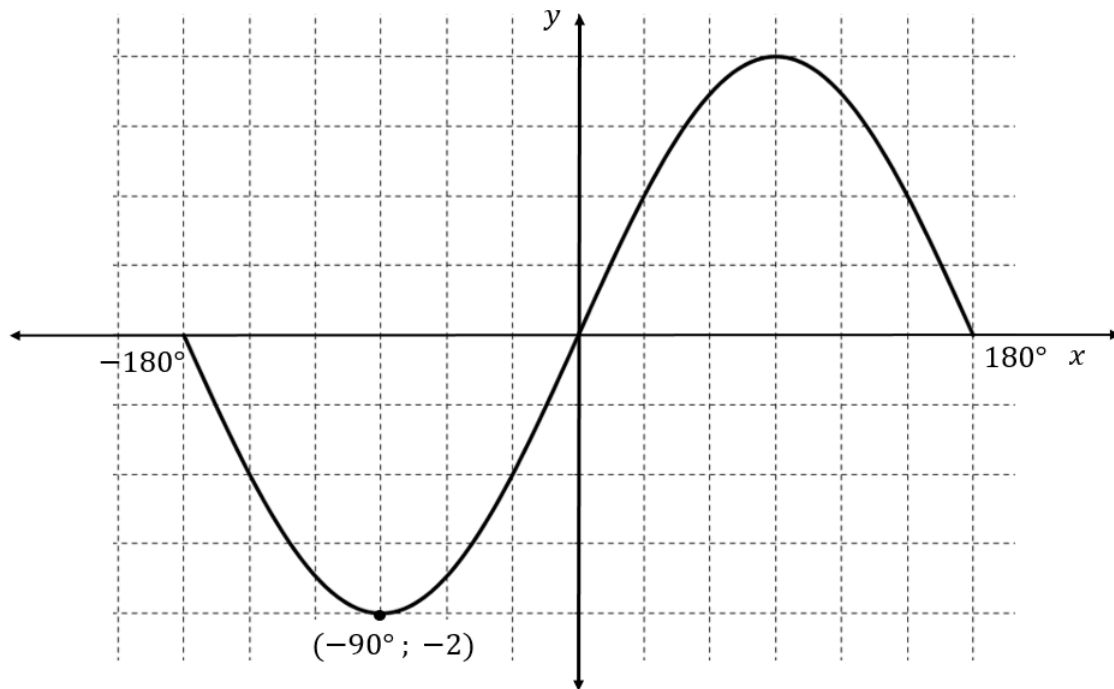
5.4.1 Prove that  $\sin A \cos B = 4 \cos A \sin B$  (3)

5.4.2 Hence, calculate the value of  $\tan A$  if  $\tan B = \frac{1}{2}$ . (3)

[23]

**QUESTION 6**

In the diagram  $g(x) = a \sin x$ ;  $x \in [-180^\circ; 180^\circ]$ .



6.1 Determine the value of  $a$ . (1)

6.2 Sketch the graph of  $f(x) = 2 \cos(x - 30^\circ)$  on the same system of axes as provided in the **answer book**. (3)

6.3 Determine the values of  $x$  if  $f(x) = g(x)$  for  $x \in [-180^\circ; 180^\circ]$ . (5)

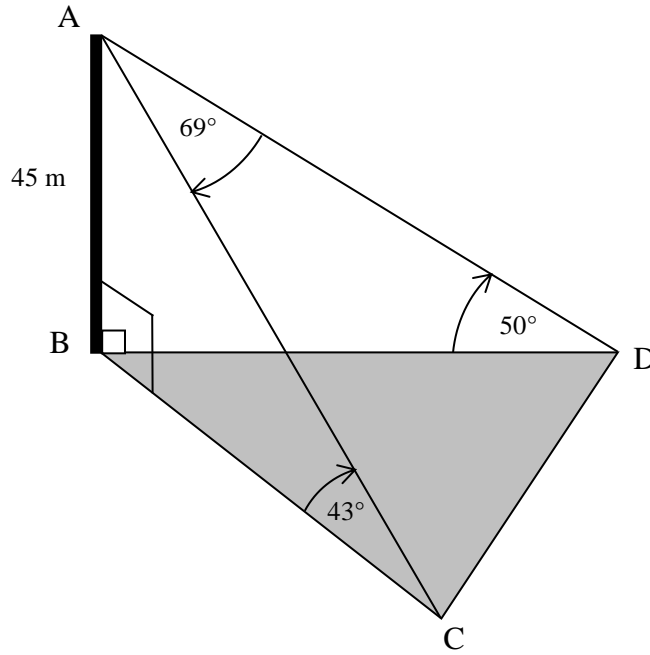
6.4 The minimum value of  $y = g(x) - k$  is equal to  $-5$ . Write down the value of  $k$ . (1)

6.5 Describe the translation from  $f$  to  $g$ . (2)

[12]

**QUESTION 7**

In the figure below Thabo is standing at a point A on top of building AB that is 45 m high. He observes two cars at C and D respectively. The cars at C and D are in the same horizontal plane as B. The angle of elevation from C to A is  $43^\circ$  and the angle of elevation from D to A is  $50^\circ$  and  $\hat{CAD} = 69^\circ$ .



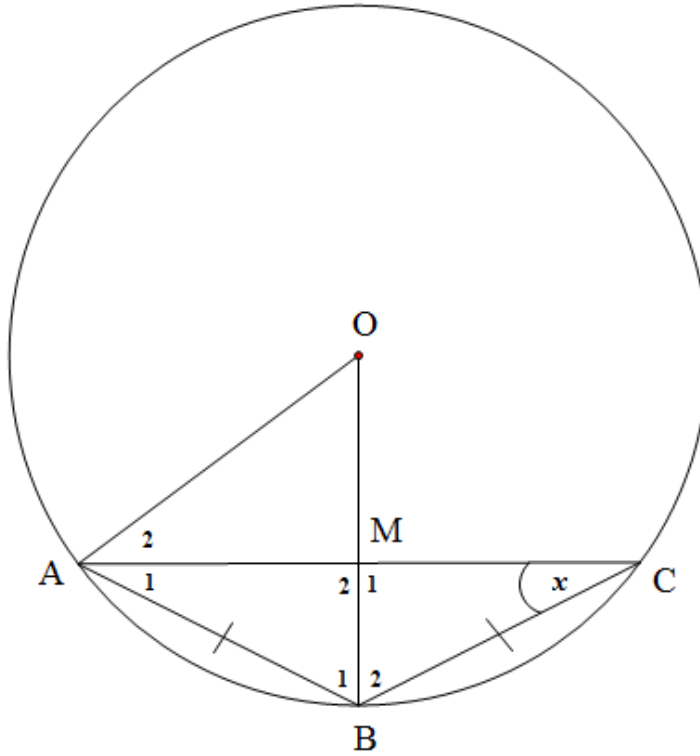
- 7.1 Calculate the lengths of AC and AD, correct to 2 decimal places. (4)
  - 7.2 Calculate the distance between the two cars, the length of CD. (3)
- [7]**



**QUESTION 8**

In the diagram below, a circle passing through A, B and C is drawn. O is the centre of the circle.

$AB = BC$  and  $\hat{ACB} = x$ .



8.1 Give, with reasons, the following angles in terms of  $x$ :

8.1.1  $\hat{A}_1$  (1)

8.1.2  $\hat{AOB}$  (2)

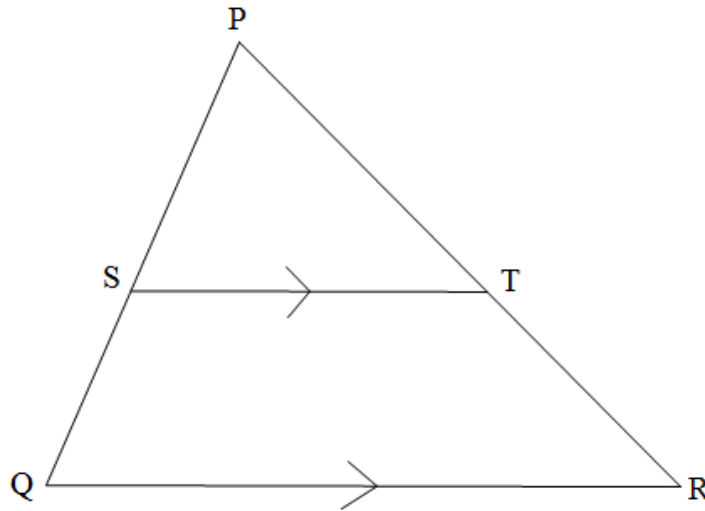
8.2 Prove that  $\hat{AMB} = 90^\circ$ . (3)

8.3 Determine the length of AM, if it is given that  $AC = 12$  units. (2)  
**[8]**



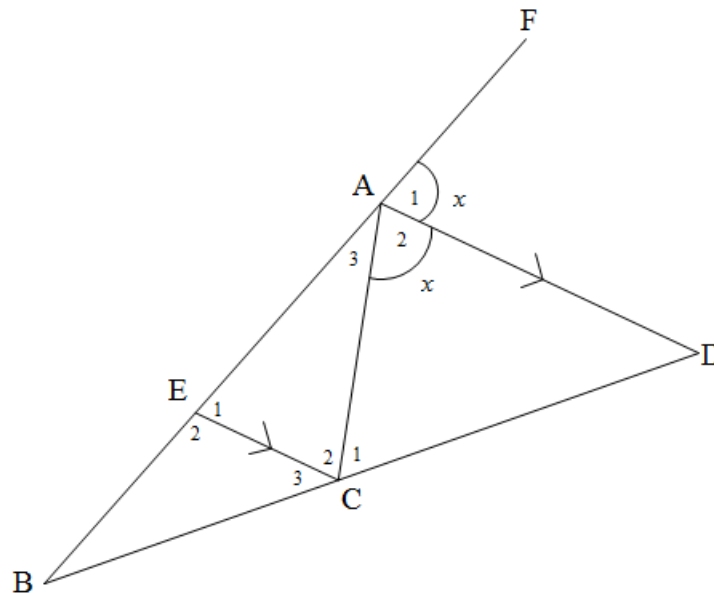
**QUESTION 10**

- 10.1 In the diagram  $\triangle PQR$  is drawn. S and T are points on sides PQ and PR respectively such that  $ST \parallel QR$ .



Prove the theorem which states that  $\frac{PS}{SQ} = \frac{PT}{TR}$ . (6)

- 10.2 In  $\triangle ABD$ , BA is produced to F. AC is drawn with C on BD such that  $\hat{A}_1 = \hat{A}_2 = x$ .  $CE \parallel DA$  with E on AB.



10.2.1 Give, with reasons, two more angles equal to  $x$ . (3)

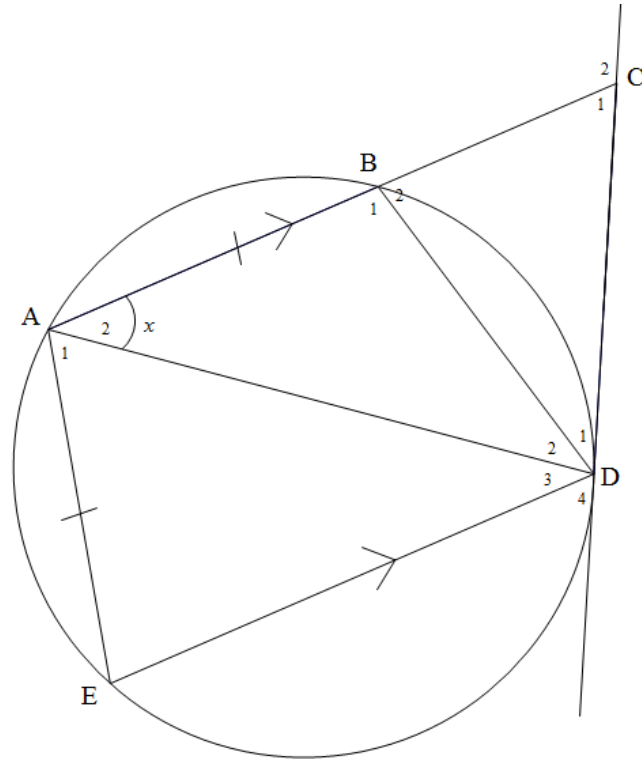
10.2.2 Prove that  $\frac{BD}{DC} = \frac{BA}{AC}$ . (4)

[13]

**QUESTION 11**

In the diagram a circle with tangent CD is given. A, B, D and E are points on the circumference of the circle.  $AE = AB$  and AB is parallel to ED.

$\hat{A}_2 = x$



- 11.1 State, with reasons, three more angles equal to  $x$ . (6)
  - 11.2 Prove that  $\triangle DEA \parallel \triangle DBC$  (4)
  - 11.3 Prove that  $BC \cdot ED = AE^2$  (5)
  - 11.4 Calculate the value of  $x$ , if it is given that  $\hat{A}_1 = 75^\circ$ . (3)
- [18]**

**TOTAL: 150**

## INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; -1 < r < 1$$

$$F = \frac{x[(1 + i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{area } \triangle ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2\sin \alpha \cdot \cos \alpha$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$